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A DEEPER LOOK AT HYPERBOLIC DISCOUNTING

ABSTRACT. We conduct an experiment to investigate the degree to which deviations from exponential discounting can be accounted for by the hypothesis of hyperbolic discounting. Subjects are asked to choose between an earlier or later payoff in a series of 40 choice questions. Each question consists of a pair of monetary amounts determined by compounding a given base amount at a constant rate per period. Two bases (8 and 20 dollars), three compounding rates (low, medium and high) and three delays (2, 4, and 6 weeks) are each used. There are also 2 initial periods (Today and 2 weeks) and there are two separate questionnaires, one with lower "realistic" compounding rates and the other with higher compounding rates, typical of those used in previous studies. We analyze the detailed patterns of choice in 6 groups of 6 related questions each (in which the base and rate is fixed but the initial period and delay varies), documenting the frequency of patterns consistent with exponential discounting and with hyperbolic discounting. We find that exponential discounting is the clear modal choice pattern in virtually all cases. Hyperbolic discounting is never the modal pattern (except in the sense that constant discounting is a special case of hyperbolic discounting). We also estimate a linear probability model that takes account of individual heterogeneity. The estimates show substantial increases in the probability of choosing the later option when the compounding rate increases, as one would expect. There are small, sometimes significant, increases in this probability when the delay is increased or the initial period is in the future. Such behavior is consistent with hyperbolic discounting, but can account for only a small proportion of choices. Overall, deviations from exponential discounting appear to be due to error, or to other effects not accounted for by hyperbolic discounting. Principal among these is an increase in later choices when the base is larger.

KEY WORDS: hyperbolic discounting, modal choice, payoff

1. INTRODUCTION

Since the late 1930s, when Samuelson introduced discounted utility, the concept of discounting has been used by economists in analyses of intertemporal choice. Although the descriptive accuracy of this model has been called into question by many, the analytic convenience and the normative logic of the model have kept it alive. In other words, it is easy to use, and it seems, for many purposes quite sensible. It is a workhorse in dynamic models in labor economics and macroeconomics, and it is, indeed, hard to imagine what model one would use in its place.

Loewenstein and Thaler (1989) summarize and review the literature illustrating many shortcomings in the model, and Loewenstein and Prelec (1992) have proposed an alternative formulation with a structure similar that proposed by Kahneman and Tversky's prospect theory for risky choice. Indeed, on closer consideration, it is often only functional form restrictions that are being questioned by these authors, rather than the fundamental notion that future utility is somehow discounted. For example, Loewenstein and Prelec (1993) pointed out that an individual might well value different sequences of restaurant meals differently, and thus they questioned the simple adding-up (with appropriate discount factors) of sequences of utility. The intuition that one might prefer to vary the cuisine of one's meals out rather than have the same thing week after week is attractive, of course, and perhaps a utility function that explicitly accounts for complementarities between adjacent periods would be more appropriate in this case.

The time period of analysis and the consumption basket that Samuelson envisioned, though, was something more like total consumption year by year. More to the point, in a dynamic model of, say, lifetime labor supply, considerations of the particulars of week by week consumption patterns is too fine a level of detail, and such models were never intended to capture such factors with perfect accuracy. The key element of the discounted utility model is, after all, not the utility function but the discounting function. We could assume risk-neutral income-maximizing behavior as the baseline behavioral model and not affect the predictions of the theory in any substantial way.

The specifics of the discounting function have, in fact, been the focus of many writers in recent years, and here there is perhaps a bit more room for improvement, even for the big-picture uses to which the discounted utility model has been put. The logical inconsistencies associated with non-constant discounting were first explored by Strotz (1955), and the tendency of some people (and rats, too) to discount in a non-constant manner have been documented in many experimental studies since then. Here, as in the criticisms of the utility function in discounted utility, a little intuition seems to go a long ways. The story, told by Thaler (1981), that I might prefer an apple today over two apples tomorrow, but that I would more likely prefer two apples in 2 weeks and a day to one apple in 2 weeks has, again, a certain appeal. Coupled with the observation that this would violate discounted utility, this is persuasion enough for some. But again, on closer examination, is it so persuasive? Will $100 today be chosen over $100 compounded at a constant daily rate tomorrow? (Surely $100 compounded at some constant rate for 15 days will be chosen over $100 compounded at the same rate for 14 days.) We suggest, provisionally, that some might violate stationarity in this way, but that most people would not.

We would suggest, further, that it is not enough that significantly fewer individuals choose the later option in the (today, tomorrow) case than in the (2 weeks, 2 weeks and a day) case to prove that the apple story is descriptively accurate. Such a result would certainly falsify the predictions of the constant discounting assumption embodied in the discounted utility model, but it does not clearly support some other clear alternative model, such as hyperbolic discounting, as some studies seem to conclude, if only implicitly. Thaler (1981), Benzion et al. (1989), Mischel (1966, 1974) Mischel and Ebbenson (1970) and Ainslie and Haendel (1983) all fall in this class. It should be pointed out as well that all of these studies made use of hypothetical payoffs only.

More specifically, most studies have focused on the two main implications of the constant discounting. The first implication is *stationarity*. Stationarity means that in a choice between two consequences, it matters only how far apart in time the consequences are delivered, and not their absolute position in time. The apple story above is meant to show how stationarity will be routinely violated. The second implication we will refer to as *linearity*. This means that if one prefers \$100 compounded at a constant rate for two 2 weeks over \$100 today, then one ought to also prefer \$100 compounded at the same constant rate for 4 weeks over \$100 today.

The studies cited above typically find that a significant proportion of subjects will violate stationarity, and a significant proportion will violate linearity as well. There is a tendency for immediate consequences to be chosen over delayed consequences, leading to the stationarity violation. There is also a tendency for later outcomes to be chosen more often, the more delayed they are. That is, later consequences seem to be discounted at a lower rate than early consequences.

Holcomb and Nelson (1992) have conducted one of the few studies that carefully tried to induce monetary incentives in the manner that experimental economists do. They found support for stationary but not linearity in their study. This analysis is a significant improvement, statistically, over previous studies, but not enough detail is provided in the analysis reported in the paper to answer the question posed above, whether the violations of constant discounting can be interpreted as, equally, support of hyperbolic discounting. In this paper, we use a design much like that of the Holcomb–Nelson experiment, but expand the design to include a larger variety of compounding rate. After making the obvious aggregate comparisons of choice frequencies between appropriate sets of questions, we then conduct two further types of analysis to more deeply probe into this question. First, we consider *patterns* of choices over large sets of questions and investigate to what degree the frequencies with which individuals choose different patterns help us to differentiate between alternative discounting schemes. Second, we treat the data as a panel of observations and conduct regression analysis, controlling for individual heterogeneity, to quantify more precisely the magnitudes of the various departures from constant discounting that we observe.

2. THE EXPERIMENT

2.1. *The discounting function*

For purposes of generating testable predictions, we will focus on the special case of discounted utility in which the utility is linear. In other words, we will focus on present value maximization as the baseline model. In this model, we suppose that individuals choose between alternative income streams in a way that maximizes present discounted value. For example, if one has a choice between $\$x$ today and $\$y = \$x(1+r)$ in 2 weeks, then one compares the present value of each alternative and chooses appropriately. That is, $x > or < y\delta$ where δ is the appropriate 2-week discount factor. The *stationarity* property of discounted utility is seen as follows. If, without loss of generality, x today is preferred to y in 2 weeks, then this means that $x \geq y\delta$. Note that the choice between x in 2 weeks and y in 4 weeks is evaluated by comparing $x\delta$ to $y\delta^2$, and, since $x\delta \geq y\delta^2$, one continues to prefer the earlier to the later option in this case. Note also that we could compound both alternatives at the same rate over the additional 2 weeks with out changing the choice pattern. That is, $x(1+r)\delta \geq y(1+r)\delta^2 = x(1+r)^2\delta^2$. Similarly, the *linearity* property of discounted utility can be seen by asking how our individual would choose between x today and $z = x(1+r)^2$ in 4 weeks. Since we know now that $(1+r)\delta \leq 1$ we also can say that $x \geq x(1+r)^2\delta^2$ so, again, the earlier option continues to be chosen.

The hyperbolic discount function is an alternative to constant discounting that has been proposed to accommodate the types of violations of constant discounting commonly observed in experimental studies. Although it is usually presented as a generalized hyperbola, a certain limit of which is equivalent to constant (exponential) discounting, we follow Holcomb and Nelson (1992) and use a discretized version of this discounting function, as it has a more natural and intuitive look. The general form of the function is $1/(1+k_1)(1+k_2)(1+k_3)\ldots(1+k_t)$ for a payment to be received t periods into the future. Moreover, we have $k_1 \geq k_2 \geq k_3 \geq \cdots \geq k_t$. Obviously, this is equivalent to constant discounting if the k's are all equal, and $1/(1+k) = \delta$, the discount factor. With this hyperbolic discounting function, both linearity and stationarity can be violated. We return to investigate this function in more detail in Section 4, when we analyze in detail the individual choice patterns in the data.

2.2. *Design*

The experiment was a survey consisting of forty choice questions. Each question offered a choice between an earlier and a later monetary payment. The payments were arrived at by applying a constant compounding rate to a given base amount. Subjects chose either the earlier or the later payment for each question. It was explained that one of the forty questions would be chosen at random at the end of the experiment, and the subject's choice on that question would be his or her payment for the experiment. If the choice involved a delay, the subject was required to return to the same room where the experiment was held on the appointed day to collect the payment. Payments varied from \$8 to more than \$40. A total of 86 subjects participated in the experiment. The latest payment was 8 weeks from the date of the experiment, which always fell with in the semester in which the experiment was conducted, so that student subjects were still on campus at the time the payment was due.

Four factors were varied between questions in the questionnaires: the base amount (8000 or 20,000 francs[1]), the compounding rate (low, medium or high), the initial time (today or in 2 weeks) and the time delay between choices (2, 4 or 6 weeks). There were two distinct questionnaires, and subjects were randomly assigned to answer one or the other, not both. The Low-Rate Questionnaire used lower compounding rates of 0.1, 0.5 and 1% per week. These translate into implied annual rates of approximately 5.2, 26 and 52.26%, respectively. The High-Rate Questionnaire used higher compounding rates of 1, 5 and 10% per week. These translate into implied annual rates of approximately 52.26, 266.5 and 546%, respectively. A total of 44 subjects answered the lower-rates questionnaire and 42 subjects answered the higher-rates questionnaire. The questionnaires are contained in the appendix.

This 2 (base) × 2 (initial period) × 3 (delays) × 3 (rates) = 36 accounts for only 36 of the 40 questions. The other four questions on each questionnaire were designed to test for simple monotonicity by asking if the subject wanted the same amount of money earlier or later, with no compounding of the initial base amount. Finally, it should be noted that order of the questions was randomized for each subject, so that no subject saw the questions in the orderly fashion shown in the appendix, where all questions for a given base amount and compounding are shown in order.

The subjects were students at Rutgers University and at New York University. The experiment was conducted in computer labs at the two institutions in the fall of 2002 and the spring of 2003. Subjects were recruited through email notification and electronic sign-up procedures. The subjects were seated at individual computer and completed the online questionnaire individually. Subjects were paid at the end of the session or given a reminder notice telling them how much they were scheduled to receive at some specified future date, and where they should come to collect their payments. All but three subjects collected their payments on schedule. Of these, two collected their payments late, and one never collected a payment.

3. AGGREGATE RESULTS

For the basic monotonicity issue (Questions 37 through 40 on both questionnaires) subjects were quite consistent. In all, 92% of the subjects (81 out of 86) chose to take the money earlier rather than later in all four questions. None of the subjects chose the later choice in all four questions. The worst violator of monotonicity chose to take the money later in three of the four questions.

We now focus on the 36 main questions that comprise the actual experiment. Table I shows the percentage of subjects choosing the delayed alternative for each question in which the initial time is today, while Table II shows the percentage choosing the delayed option when the initial period is 2 weeks from today. There is a clear increase in the number of people choosing the delayed option as the interest rate increases, as one would expect. There is also some evidence, though not so obvious and uniform, that more individuals choose the delayed option as the delay grows bigger. If this is a systematic result, then this is a violation of the linearity property of constant discounting.

The rates at which subjects choose the more delayed alternative in Table II are broadly consistent with those in Table I, though in many cases the rates are higher in Table II than the corresponding entries in Table I, suggesting that the stationarity property of constant discounting is often violated.

Figures 1 and 2 help us to visualize the data in Tables I and II. The figures show essentially the same information as in the tables, except that the high and low-base questions are aggregated for each interest rate/initial period/delay combination. The figures show the percentage choosing the later option on the vertical axis, and the delay from today of the later option in each pair. The three averages for a given interest rate and initial period are connected by straight line segments. As a point of reference, if the subjects used constant discounting unfailingly and without error, the two sets of line segments corresponding to each interest rate should coincide and form a perfectly horizontal line. Instead, there is a tendency for the line segments associated with the choice questions where the earliest period was also delayed to lie above the segments associated with the choice questions where the earliest period was today. This illustrates the extent to which stationarity is violated. Also, there is some tendency for the line segments to slope upwards, especially when the initial period is today. This illustrates the extent to which linearity is violated.

TABLE I
Percentage choosing more delayed alternative for initial time = today

	Lower-rates questionnaire		Higher-rates questionnaire	
	High base	Low base	High base	Low base
Today versus 2 weeks from today				
Low rate	9.09	11.36	21.43	16.67
Medium rate	43.18	20.45	40.48	33.33
High rate	50.00	45.45	52.38	57.14
Today versus 4 weeks from today				
Low rate	15.91	15.91	16.67	19.05
Medium rate	40.91	34.09	54.76	50.0
High rate	54.55	50.00	80.95	64.29
Today versus 6 weeks from today				
Low rate	18.18	18.18	21.43	16.67
Medium rate	38.64	40.91	59.52	54.76
High rate	56.82	56.82	88.10	66.67

The conventional wisdom that both stationarity and linearity are violated receive some support from the aggregate averages, but it is not clear if the departures from constant discounting are limited to a distinct set of individuals, or if it is a general phenomenon. We now turn to analysis of the data in which we take careful account of individual patterns of choice behavior.

4. CHOICE PATTERNS

Each questionnaire has 36 questions with varying bases and rates (the last four questions are to test for monotonicity and

TABLE II

Percentage choosing more delayed alternative for initial time = 2 weeks from today

	Lower-rates questionnaire		Higher-rates questionnaire	
	High base	Low base	High base	Low base
2 weeks from today versus 6 weeks from today				
Low rate	25.00	29.55	23.81	28.57
Medium rate	43.18	43.18	45.24	50.00
High rate	54.55	54.55	66.67	64.29
2 weeks from today versus 6 weeks from today				
Low rate	36.36	22.73	26.19	11.90
Medium rate	47.73	52.27	42.86	47.62
High rate	65.91	50.00	90.48	71.43
2 weeks from today versus 8 weeks from today				
Low rate	25.00	22.73	33.33	14.29
Medium rate	47.73	40.91	61.90	30.95
High rate	56.82	50.00	85.71	71.43

so are ignored in this analysis). There are three rates (high, medium and low) and two bases (high and low). We form six sets of six questions. Each set of six questions corresponds to a given base and compounding rate.

There are $2^6 = 64$ possible choice patterns for each set of six questions. Patterns 1 and 64 correspond to constant discounting and are, in fact, the only "legal" choice patterns one should see if individuals discount at a constant rate. We have enumerated all of the possible patterns[2] and computed

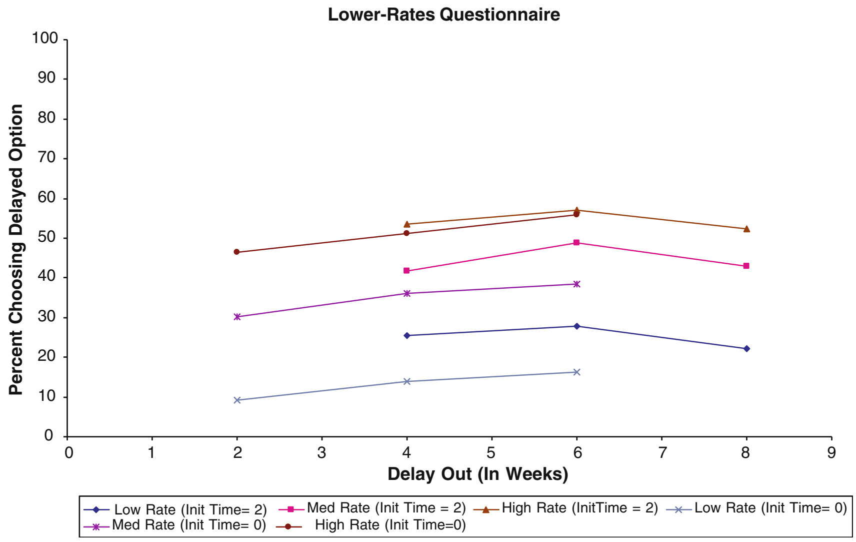

Figure 1. Increasing the delay in the lower-rates questionnaire for the average of both the bases.

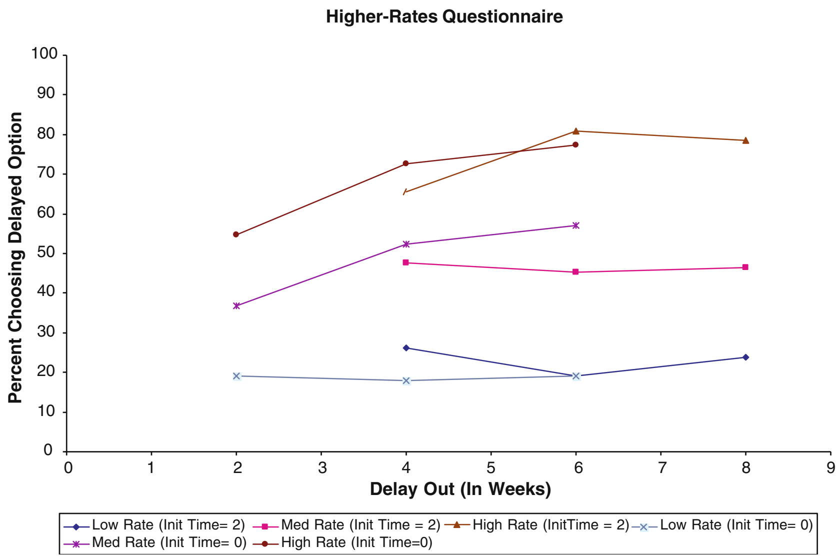

Figure 2. Increasing the delay in the higher-rates questionnaire for the average of both the bases.

the distribution of choice patterns for each of the six sets of six questions. The numbering of the patterns corresponds roughly to the number of late choices: the larger the number, the greater the number of late choices in the pattern.

Table III illustrates the eight choice patterns that are consistent with hyperbolic discounting. A choice pattern may be represented by 6 indicator variables, specifying whether the earlier (0) or later (1) choice was made on each question. The columns in the matrices have a common initial time: Each indicator in column 1 corresponds to a question where the initial time is today (0 weeks) and each member of column 2 has an initial time of 2 weeks. The rows in the matrices have a common delay: Each indicator in row 1 has a delay of 2 weeks, each indicator in row 2 has a delay of 4 weeks and each indicator in row 3 has a delay of 6 weeks. Thus, for example, Pattern 1 is the pattern in which all choices are early choices, and Pattern 64 is the pattern in which all choices are late choices. The other six patterns shown have various combinations of early and late choices, but each can be rationalized as a possible "strictly hyperbolic" pattern for some set of k_i's. Note that any pattern in which there is a 1 above a 0 in a column, or any row with a 1 to the left of a 0, cannot be a hyperbolic pattern. No other patterns can be rationalized as consistent with either constant discounting or strict hyperbolic discounting.[3]

The logic of the experimental design was to observe the change in behavior of the subject as the delay or the initial time increases. In each of the patterns, as one goes down a particular column, the delay is increasing. Furthermore, as one switches from the left column to the right column in any row, the initial time period is being pushed out. An individual falling into Pattern 7, for example, chooses the delayed alternative in the sixth question — the one which has the longest delay (6 weeks) and an initial time of 2 weeks. This individual shows characteristics we would expect in a hyperbolic type; that is, if the initial time and delay are pushed out enough, then the subject's effective discount rate (which is determined by the k_is) falls to a point below the compounding rate in the

TABLE III
Hyperbolic choice pattern definitions

$0 \equiv$ early and $1 \equiv$ late.

Pattern 1: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$	Pattern 7: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$	Pattern 22: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$	Pattern 41: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$
Pattern 42: $\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$	Pattern 56: $\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$	Pattern 63: $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	Pattern 64: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

sixth question. As a consequence, he or she will choose the later option. An individual of Pattern 22 would choose the later option in the fifth and sixth questions, for an initial time of 2 weeks for a delay of either 4 or 6 weeks.

In general, each hyperbolic pattern implies a particular configuration for the way that the k_is decline, and it is difficult (and perhaps pointless) to try to isolate or infer the precise pattern. The generalized hyperbola (a special case being the exponential) was shown by Loewenstein and Prelec (1992) to be the only possible form for the discounting function that will permit both violations of stationarity and linearity. Such a discounting function is a "smooth" convex function in discount rate-time space. For the discretized version of the discounting function that we have adopted, we determined which patterns were consistent with hyperbolic discounting simply by showing that there is some nonincreasing sequence of $k_i, i = 1, 2, 3, 4$, that allows one to rationalize the pattern. It is worth considering in more detail whether any such sequence is necessarily consistent with a generalized hyperbola (see Table IV).

In this way, as one progressively goes from Pattern 7 to Pattern 63, the k_i's are progressively getting smaller. For example, for pattern 7, we can infer that $k_1 > r$, that $k_2 > r$, that (redundantly, and roughly speaking) the average of k_1 and k_2 is greater than r, that the average of k_1, k_2 and k_3, and the average of k_2 and k_3 are each bigger than r, but that the average of k_1, k_2, k_3, and k_4 is less than r. In other words, k_4 is sufficiently small to bring the average discount factor down far enough so that the later choice is made for a sufficiently delayed payment. At the other extreme, in pattern 63, all but the first of the conditions have been reversed. That is, we still infer that $k_1 > r$, but k_2, k_3 and k_4 are sufficiently small, on average and independently, so that all but one choice is the later choice. For patterns in between, Table IV maps out the inequalities that must hold.

5. DISTRIBUTIONS OF CHOICE PATTERNS

How should one interpret the observed choice patterns? One possibility is to take the observed choice pattern as a genuine and perfectly accurate reflection of the subject's preferences. In this case, one naturally would want to look at what happens to the observed distribution of patterns as the compounding rate is increased. The hypothesis of constant discounting leads one to posit that there is some threshold of the compounding rate at which individuals switch from all early to all late choices. The hypothesis of hyperbolic discounting is a little tricky (since there is not a single threshold), but still relatively straightforward. As the compounding rate is increased one would expect to see plenty of "all early" patterns at low rates and plenty of "all late" patterns with, one would hope, some strict hyperbolic patterns in between. One might also find strict hyperbolic patterns at low rates (presumably followed by "all late" patterns at medium and higher rates), or "all early" patterns for low and medium rates, with strict hyperbolic patterns at higher rates.

More realistically, one has to allow for the possibilities that there is a stochastic element in the choices subjects make. Indeed, we have given the subjects plenty of rope to hang

TABLE IV

Conditions on the discounting factors in the hyperbolic function implied by various choice patterns

Pattern	Conditions on k_1	Conditions on k_1, k_2	Conditions on k_1, k_2, k_3	Conditions on k_1, k_2, k_3, k_4
7	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) > (1+r)^2$ and $(1+k_2) > (1+r)$	$\Pi_{i=1}^{3}(1+k_i) > (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) > (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$
22	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) > (1+r)^2$ and $(1+k_2) > (1+r)$	$\Pi_{i=1}^{3}(1+k_i) > (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) < (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$
41	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) > (1+r)^2$ and $(1+k_2) > (1+r)$	$\Pi_{i=1}^{3}(1+k_i) < (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) < (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$
42	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) > (1+r)^2$ and $(1+k_2) < (1+r)$	$\Pi_{i=1}^{3}(1+k_i) > (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) < (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$
56	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) > (1+r)^2$ and $(1+k_2) < (1+r)$	$\Pi_{i=1}^{3}(1+k_i) < (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) < (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$
63	$(1+k_1) > (1+r)$	$\Pi_{i=1}^{2}(1+k_i) < (1+r)^2$ and $(1+k_2) < (1+r)$	$\Pi_{i=1}^{3}(1+k_i) < (1+r)^3$ and $\Pi_{i=2}^{3}(1+k_i) < (1+r)^2$	$\Pi_{i=1}^{4}(1+k_i) < (1+r)^3$

TABLE V
Distribution of possible choice patterns

# Late choices	# of Patterns	Patterns #s	Hyperbolic discounting?
No late choices	1	Pattern 1	1
1 Late choice	6	Patterns 2–7	7
2 Late choices	15	Patterns 8–22	22
3 Late choices	20	Patterns 23–42	41, 42
4 Late choices	15	Patterns 43–57	56
5 Late choices	6	Patterns 58–63	63
6 Late choices	1	Pattern 64	64

themselves, so to speak. There are only 2 patterns out of 64 consistent with constant discounting, and 6 others consistent with strict hyperbolic discounting (though, as noted above, a strictly hyperbolic type may choose constant discounting patterns for sufficiently high or low compounding rates). This leaves 56 of 64 patterns as clear mistakes or errors, even under the rather generous allowances that the hyperbolic discounting hypothesis provides. In order to begin to get a feel for the data, Tables V and VI show the set of possible patterns that one might observe, and the distributions of the actual choices made, for each base level and compounding rate. We have combined the results from the low-rate and high-rate questionnaires for these tables, as the same point could be made for each questionnaire separately or together, even though the distributions are not identical. In later regression analysis we will treat these results separately.

As Table V shows, for each set of 6 related choice questions, there are $\binom{6}{n}$ patterns involving n late choices, adding up to 64 possible patterns in all. Table V also indicates which numbered patterns correspond to patterns with the various number of late choices. The most notable feature of Table VI,

TABLE VI
Distributions of actual choice patterns

Low compound rate				Medium compound rate				High compound rate			
Low base		High base		Low base		High base		Low base		High base	
Pattern #/Frequency											
1	56	1	45	1	29	1	22	1	17	1	16
4	1	2	1	4	1	2	1	2	1	8	1
5	6	3	1	5	3	3	1	3	1	13	1
7	1	4	3	6	2	4	2	4	1	22	1
8	1	5	3	7	1	5	1	5	2	32	1
10	1	6	2	8	1	7	3	6	2	33	1
13	1	7	3	9	2	8	1	7	1	38	1
17	2	8	1	13	2	9	1	8	1	39	2
20	1	15	1	17	2	10	1	9	1	41	4
21	1	20	2	18	1	12	1	10		42	1
22	2	22	5	20	3	13	2	17	1	44	1
23	1	26	1	22	1	15	1	20	1	46	1
24	2	28	1	32	1	19	2	21	1	49	1
35	1	40	1	33	1	20	2	22	2	55	5
42	3	41	1	34	2	21	1	23	1	56	1
48	1	42	4	42	2	22	2	34	1	57	2
53	1	53	1	44	1	24	1	41	2	58	1
58	1	56	1	46	1	35	2	42	1	59	1
59	1	64	9	47	1	42	3	43	2	60	2
62	1			53	2	43	1	46	1	62	1
63	2			55	1	44	2	48	1	63	7
64	3			56	2	45	1	55	3	64	34
				58	2	47	1	56	3		
				60	1	55	2	59	3		
				63	8	56	2	63	3		
				64	13	59	2	64	32		

TABLE VI
Continued

Low compound rate		Medium compound rate		High compound rate	
Low base	High base	Low base	High base	Low base	High base
Pattern #/Frequency					
			60 2		
			61 2		
			62 1		
			63 2		
			64 18		

keeping in mind the information contained in Table V, is the wide divergence between the expected frequency of patterns of each sort and the actual. Table VII summarizes this information. What is striking is that the frequency (proportion) of patterns with 1, 2, 3, 4 or 5 late choices is approximately equal, at about 0.09, on average. A second thing to note in Table VI is that the frequency of observed hyperbolic patterns for a given number of late choices (1 to 5) is generally higher than one would expect if the patterns were purely random error. This is not so unreasonable, as the hyperbolic patterns, unlike the other patterns, do in fact satisfy some basic dominance properties. This is also summarized in Table VII.

A provisional interpretation of all of this is that, while the absolute frequency of non-constant discounting patterns is non-trivial (44% of all choices), the proportion of these patterns that are hyperbolic is only about a third (34%), implying that only about 15% of all choices are hyperbolic patterns, which is not such strong evidence for hyperbolic discounting as an overall account for intertemporal choice behavior. On the other hand, the last column of Table VII suggests that deviations from constant discounting are not purely random errors, and are somewhat inclined to be driven by the sorts

TABLE VII

Summary information about actual proportions of patterns according to number of later choices

Number of late choices	Expected proportion	Actual proportion (average over all cases) (516 cases in all)	Expected proportion of hyperbolic patterns within category	Actual Proportion of hyperbolic patterns in each category
0	1/64=0.02	185/516=0.36	1.0	185/185=1.0
1	6/64=0.09	44/516=0.09	0.17	9/44=0.20
2	15/64=0.23	47/516=0.11	0.07	13/47=0.28
3	20/64=0.31	42/516=0.08	0.10	21/42=0.50
4	15/64=0.23	42/516=0.08	0.07	9/42=0.21
5	6/64=0.09	43/516=0.08	0.17	22/43=0.51
6	1/64=0.02	109/516=0.21	1.0	109/109=1.0

of factors that hyperbolic discounting is meant to account for: stationarity violations and linearity violations.

Up to now the analysis has been descriptive and has dealt in aggregate behavior. We now proceed to conduct regression analysis that will allow us to formally account for individual-specific factors, and to formally quantify the effects of extending the time horizon and of shifting all payment periods.

6. REGRESSION ANALYSIS OF THE PROBABILITY OF CHOOSING LATER OPTIONS

In order to separate the effects of differences in individual propensities to choose later choices from the effects of such things as the length of time between payments and the placement in time of the payment, we organize the data as a panel of observations and take specific account of individual heterogeneity. Individuals may have different propensities to choose later choices due to higher or lower average subjective[4] discount rates. A linear probability model (LPM) for a binary response y may be specified as

$$P(y=1|\mathrm{x}) = \beta_0 + \beta_1 x_1 + \cdots + \beta_K x_K \tag{6.1}$$

where $P(y=1|\mathbf{x})$ is the probability that the later of the two choices in a question is chosen. That is, the event that the later choice is chosen is coded as $y=1$, and otherwise $y=0$. Assuming that x_i is not functionally related to the other explanatory variables, $\beta_i = \partial P(y=1|\mathbf{x})/\partial x_i$. Therefore, β_i is the change in the probability of success given a one-unit increase in x_i. If x_i is a binary explanatory variable (as it always will be in our analysis), then β_i is just the difference in the probability of success when $x_i=0$ and $x_i=1$, holding the other x_j fixed. Since all of the regressors are 0–1 variables, our analysis is not vulnerable to one of the usual criticisms made of the linear probability model, that the fitted values for P may be larger than 1 or less than 0.

Another advantage of using the linear probability model, instead of some nonlinear transformation function, such as the logit or probit, is that it is straightforward to allow for individual heterogeneity in choice behavior. We estimate the model by specifying the error term as $u_{\mathrm{im}} = e_{\mathrm{im}} + c_i$. That is, the error is modeled as being the sum of an individual specific component c_i and an idiosyncratic component e_{im} that varies from observation to observation on an individual. We use a random effects specification in the estimation.

$$P(y=1|\mathrm{x}, c_i) = \beta_0 + \beta_1 x_1 + \cdots + \beta_K x_K + c_i + e_{\mathrm{im}} \tag{6.2}$$

The regressors are:
ratem = 1 if compounding rate is the medium rate, 0 otherwise
rateh = 1 if compounding rate is the high rate, 0 otherwise
delay 4 = 1 if the delay between the earlier and later payment is 4 weeks, 0 otherwise
delay 6 = 1 if the delay between the earlier and later payment is 6 weeks, 0 otherwise

initial $2 = 1$ if the initial period (when the earlier payment is made) is 2 weeks, 0 otherwise

baseh $= 1$ if the base amount is the higher level ($20), 0 otherwise (if it is $8).

To summarize, the estimated coefficients on these variables in the linear probability model indicate the marginal increase or decrease in the probability of choosing the later option relative to the probability of choosing earlier when the initial period is today, the compounding rate is low, the delay between payments is 2 weeks, and the base amount is $8. We estimate these effects separately for the lower-rate and the higher-rate questionnaires. We estimate the linear probability model using generalized least squares with random effects. The estimation results are reported in Tables VIII and IX.

The results in Table VIII for the lower-rates questionnaire may be interpreted as follows. The "baseline" probability of choosing the later option in the first choice question with the low compounding rate, the low base amount, the early payment today and the later payment in two weeks, is 0.09 (the constant term). The medium rate raises this probability by 0.20, the high rate by 0.33, as one would expect (i.e., the change should be positive and non-trivial). A delay of either four or six weeks (compared to two weeks) raises this probability by about the same amount, roughly 0.05. Shifting the initial payment time to two weeks raises this probability by 0.08. These last two effects are the effects that the hyperbolic hypothesis is meant to accommodate. Raising the base amount also has a significant effect, although it is quite small in magnitude (less than 0.005). The results in Table IX for the higher-rates questionnaire are qualitatively similar.

The violations of stationarity and linearity implied by the coefficients on Initial2 and the Delay variables here are significant, but not huge. The linearity effect does not seem to be too strong after the initial increase from a 2-week delay to a 4-week delay, as shown by the similarity of the coefficients on the two delay variables (especially in Table VIII). The stationarity effect ranges from 0.03 to 0.08, implying an increase in later choices of 3–8% when the initial payment period is shifted out

TABLE VIII

Random effects LPM analysis of the lower-rates questionnaire

R^2 within=0.14		Wald $X^2(6)=259.11$	Number of obs.=1584
R^2 between=0.00		Prob> $X^2=0.00$	Number of groups=44
Overall $R^2=0.09$			Obs. Per group=36
Variable	Coefficient	z-statistic	$P>\|z\|$
Ratem	0.20	9.21	0.00
Rateh	0.33	14.98	0.00
Delay 4	0.05	2.15	0.03
Delay 6	0.04	1.64	0.10
Initial 2	0.08	4.57	0.00
Baseh	0.00	2.18	0.03
Constant	0.09	1.72	0.09
$\sigma_c=0.30$			
$\sigma_e=0.36$		$\rho=0.42$ (fraction of var. due to c_i)	.

in time. Overall, the results are consistent with what we have already observed: violations of constant discounting are widespread, but the degree to which they can be accounted for by the hypothesis of hyperbolic discounting is modest.

One further bit of regression analysis that we think helps to organize the data is motivated by the discussion in Section 5 about the frequency of choice patterns with various numbers of late choices. If we organize the data so that the unit of observation is a set of six related choice questions, and the "choice" observed is the number of later choices made in those six questions, rather than the simple binary choice of earlier or later on each question individually, then we can estimate the probability that the number of late choices per set of questions will occur. We divide the 36 choice questions into six sets of six questions each. The only variation over these sets is in the base amount and the compounding rate. That is, for each group of six questions the base amount and compounding rate is the same. The initial period and delays are subsumed within each set, so no effect of these variables can be estimated in this instance. This analysis, then, can be thought of as quantifying the pure effect of the compounding rate and the base amount, once individual-specific effects are accounted for.

There are seven possible "responses" for each such set of six questions. We simply record the number of late choices in each set, regardless of whether the patterns are hyperbolic or not, and without regard to which specific questions had an early or late response. Responses with n late choices, $n = 0, \ldots, 6$, are coded as n. We then estimate a random effects ordered probit model.[5] "Cut-points" (essentially separate constant terms for each instance of n) as well as coefficient estimates for the base amount and indicator variables for the different compounding rates are the output of the estimation. Random effects are assumed, meaning, as in the LPM estimation, that we are allowing for an individual-specific error component that is fixed over all six observations on an individual.

Tables X and XI contain estimates from the ordered probit procedure, and Table XII contains the fitted (predicted) values for the probabilities of each category (0 through 6 late choices in a six-choice set). The probabilities, for a given base value and compounding rate, are calculated as follows:

Probability of 0 Late Choices $= \Phi(c_0 - b - r)$
Probability of 1 Late Choice $= \Phi(c_1 - b - r) - \Phi(c_0 - b - r)$
Probability of 2 Late Choice $= \Phi(c_2 - b - r) - \Phi(c_1 - b - r)$
Probability of 3 Late Choice $= \Phi(c_3 - b - r) - \Phi(c_2 - b - r)$
Probability of 4 Late Choice $= \Phi(c_4 - b - r) - \Phi(c_3 - b - r)$
Probability of 5 Late Choice $= \Phi(c_5 - b - r) - \Phi(c_4 - b - r)$
Probability of 6 Late Choices $= 1 - \Phi(c_5 - b - r)$

In these calculations, Φ is the standard normal cumulative distribution function, $c_i i = 0, \ldots, 5$ are the estimated cut-points, and b and r stand for the estimated coefficients on dummy variables for the specific base amount and compounding rate in question.[6] In the estimates reported in Tables X

TABLE IX

Random effects LPM analysis of the higher-rates questionnaire

R^2 within=0.27		Wald $X^2(6) = 552.31$	Number of obs.=1512
R^2 between=0.00		Prob> $X^2 = 0.00$	Number of groups=42
Overall $R^2 = 0.19$			Obs. Per group=36
Variable	Coefficient	z-statistic	$P > \|z\|$
Ratem	0.27	11.96	0.00
Rateh	0.51	22.68	0.00
Delay 4	0.06	2.83	0.01
Delay 6	0.09	3.90	0.00
Initial 2	0.03	1.59	0.11
Baseh	0.01	4.34	0.00
Constant	0.05	0.96	0.34
$\sigma_c = 0.28$			
$\sigma_e = 0.36$		$\rho = 0.38$ (fraction of var. due to c_i)	

TABLE X

Ordered probit estimates, lower-rates questionnaire

Likelihood. ratio $X^2(3) = 96.80$	Prob> $X^2 = 0.00$	$N = 264$	Log likelihood = −335.39
Variable	Coefficient	z-statistic	$P > \|z\|$
Baseh	0.28	1.75	0.08
Ratem	1.35	6.34	0.00
Rateh	2.13	9.13	0.00
Cut1	0.87	3.92	0.00
Cut2	1.31	5.83	0.00
Cut3	2.07	8.58	0.00
Cut4	2.56	10.10	0.00
Cut5	3.00	11.24	0.00
Cut6	3.53	12.29	0.00
ρ	0.85	30.52	0.00

TABLE XI

Ordered probit estimates, higher-rates questionnaire

Likelihood. ratio $X^2(3) = 183.89$	Prob> $X^2 = 0.00$	$N = 252$	Log likelihood $= -326.05$
Variable	Coefficient	z-statistic	$P > \|z\|$
Baseh	0.50	3.10	0.00
Ratem	1.84	8.19	0.00
Rateh	3.26	11.84	0.00
Cut1	0.94	4.23	0.00
Cut2	1.67	7.24	0.00
Cut3	2.21	9.13	0.00
Cut4	2.70	10.28	0.00
Cut5	3.36	11.25	0.00
Cut6	4.11	12.17	0.00
ρ	0.81	22.84	0.00

and XI, Ratem and Rateh are the dummies for the medium and high rates, respectively, and Baseh is the dummy for the high base amount. The estimated coefficients in Tables X and XI are difficult to interpret; the fitted values in Table XII provide a more intuitive picture of the experiment. Several things are notable. First, we see now (unlike in Table VII, where all of the treatments were pooled), how there is a movement towards patterns with a larger number of late choice as the base amount increases and as the compounding rate increases (both within a questionnaire, and between the two questionnaires). Second, there is a remarkable amount of inertia in the choices: large increases in the compounding rate lead to smaller shifts towards late choices than one would expect if people were doing constant discounting. We already know from our earlier analysis that most of these "in between" choices (neither all early or all late within a set) are not, in fact, hyperbolic choices.

These estimation results should not be taken too seriously (say, as a forecasting model), but they are suggestive. There is a

TABLE XII

Estimated probabilities of choosing *n* late choices out of six

Lower-rates questionnaire

N	Base=$8, Rate=.001	Base=$8, Rate=.005	Base=$8, Rate=.01	Base=$20, Rate=.001	Base=$20, Rate=.005	Base=$20, Rate=.01
0	0.81	0.32	0.10	0.72	0.22	0.06
1	0.10	0.17	0.10	0.13	0.15	0.07
2	0.08	0.28	0.27	0.11	0.30	0.23
3	0.01	0.12	0.19	0.03	0.15	0.19
4	0.00	0.06	0.14	0.01	0.09	0.16
5	0.00	0.04	0.11	0.00	0.06	0.15
6	0.00	0.01	0.08	0.00	0.03	0.13

Higher-rates questionnaire

N	Base=$8, Rate=.01	Base=$8, Rate=.05	Base=$8, Rate=.1	Base=$20, Rate=.01	Base=$20, Rate=.05	Base=$20, Rate=.1
0	0.86	0.18	0.01	0.67	0.08	0.00
1	0.13	0.25	0.05	0.21	0.17	0.02
2	0.03	0.23	0.09	0.08	0.20	0.04
3	0.01	0.16	0.14	0.03	0.19	0.08
4	0.00	0.13	0.25	0.01	0.20	0.20
5	0.00	0.05	0.27	0.00	0.12	0.30
6	0.00	0.01	0.20	0.00	0.04	0.36

strong assumption implicit in the ordered formulation that the categories coded as “larger” and “smaller” lie in some natural ordering. If the categories between 0 and 6 later choices are, in fact, truly errors, then the ordered formulation may well overstate the degree to which those categories are likely to be chosen, especially in making predictions via the fitted values of the choice probabilities. The fitted values tend to be “smeared” over more categories than, in fact, were seen to be chosen in the raw data. This is particularly evident in the high-rates questionnaire results for the high compounding rate.

Something we have not yet remarked upon but which is quite clear, even in the summary statistics in Tables I and II, is that subjects make choices largely based upon the relative magnitudes of the compounding rates that they (implicitly) face. Although the highest compounding rate in the lower-rate questionnaire is the same as the lowest rate in the higher-rate questionnaire, the patterns of choice are only very slightly skewed towards more late choices in the higher rates questionnaire. It is not clear that this presents a major problem for the theory of intertemporal choice, though it surely does cast doubt upon the notion that the discount rate is a hard-wired part of an individual's preference structure. In particular, one cannot with confidence forecast choices for a given compounding rate when choice behavior is evidently so context-dependent. A better account may be that intertemporal preferences are constructed from the context in which one is choosing. In "real life," one is, however vaguely, aware of the options available, and tries to choose the best option, with a variety of constraints in place. In an artificial experimental setting, though the money is quite real, the options vary more widely than in the natural setting, and there is, perhaps, a tendency to try to establish what is "usual" or "normal" in the context of the experiment. Nonetheless, to the extent that subjects settle upon a notion of what is more and less preferred, even if it is context-dependent, the results may be perfectly reliable as an indicator of what people do in other naturally-occurring environments.

7. CONCLUSIONS

Our initial motivation for conducting this experiment was to try to quantify more precisely the degree to which violations of constant discounting, which we accept to be common and pervasive, can be accounted for by the hypothesis that individuals use hyperbolic discounting. We have approached this question in a number of ways: comparisons of the raw averages of choice frequencies, detailed examination of the choice patterns, a linear probability model, accounting for individual-specific effects, and an exploratory estimation of an ordered-probit formulation, also accounting for individual-specific effects. To summarize the results, without restating them, we can say that the absolute magnitude of the evidence supporting the hyperbolic discounting hypothesis is rather small. We suggest, provisionally, that a better account of the data may lie in thinking more generally of propensities to choose earlier or later that are stochastic, and that result in choice patterns that are nearer to constant discounting, the stronger are the factors that influence these propensities. Put differently, it may be better to try to come up with a plausible statistical account of the observed behavior than to enshrine what may be, after all, just a collection of biases, into a formal theoretical account of intertemporal choice behavior.

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APPENDIX I: QUESTIONNAIRES

Higher-Rates Questionnaire

1 Which do you prefer, 8000 francs in 0 weeks, or 8161 francs in 2 weeks?
2 Which do you prefer, 8000 francs in 0 weeks, or 8325 francs in 4 weeks?
3 Which do you prefer, 8000 francs in 0 weeks, or 8492 francs in 6 weeks?

4 Which do you prefer, 8161 francs in 2 weeks, or 8325 francs in 4 weeks?
5 Which do you prefer, 8161 francs in 2 weeks, or 8492 francs in 6 weeks?
6 Which do you prefer, 8161 francs in 2 weeks, or 8663 francs in 8 weeks?
7 Which do you prefer, 20000 francs in 0 weeks, or 20402 francs in 2 weeks?
8 Which do you prefer, 20000 francs in 0 weeks, or 20812 francs in 4 weeks?
9 Which do you prefer, 20000 francs in 0 weeks, or 21230 francs in 6 weeks?
10 Which do you prefer, 20402 francs in 2 weeks, or 20812 francs in 4 weeks?
11 Which do you prefer, 20402 francs in 2 weeks, or 21230 francs in 6 weeks?
12 Which do you prefer, 20402 francs in 2 weeks, or 21657 francs in 8 weeks?
13 Which do you prefer, 8000 francs in 0 weeks, or 8820 francs in 2 weeks?
14 Which do you prefer, 8000 francs in 0 weeks, or 9724 francs in 4 weeks?
15 Which do you prefer, 8000 francs in 0 weeks, or 10721 francs in 6 weeks?
16 Which do you prefer, 8820 francs in 2 weeks, or 9724 francs in 4 weeks?
17 Which do you prefer, 8820 francs in 2 weeks, or 10721 francs in 6 weeks?
18 Which do you prefer, 8820 francs in 2 weeks, or 11820 francs in 8 weeks?
19 Which do you prefer, 20000 francs in 0 weeks, or 22050 francs in 2 weeks?
20 Which do you prefer, 20000 francs in 0 weeks, or 24310 francs in 4 weeks?
21 Which do you prefer, 20000 francs in 0 weeks, or 26802 francs in 6 weeks?

22 Which do you prefer, 22050 francs in 2 weeks, or 24310 francs in 4 weeks?
23 Which do you prefer, 22050 francs in 2 weeks, or 26802 francs in 6 weeks?
24 Which do you prefer, 22050 francs in 2 weeks, or 29549 francs in 8 weeks?
25 Which do you prefer, 8000 francs in 0 weeks, or 9680 francs in 2 weeks?
26 Which do you prefer, 8000 francs in 0 weeks, or 11713 francs in 4 weeks?
27 Which do you prefer, 8000 francs in 0 weeks, or 14172 francs in 6 weeks?
28 Which do you prefer, 9680 francs in 2 weeks, or 11713 francs in 4 weeks?
29 Which do you prefer, 9680 francs in 2 weeks, or 14172 francs in 6 weeks?
30 Which do you prefer, 9680 francs in 2 weeks, or 17149 francs in 8 weeks?
31 Which do you prefer, 20000 francs in 0 weeks, or 24200 francs in 2 weeks?
32 Which do you prefer, 20000 francs in 0 weeks, or 29282 francs in 4 weeks?
33 Which do you prefer, 20000 francs in 0 weeks, or 35431 francs in 6 weeks?
34 Which do you prefer, 24200 francs in 2 weeks, or 29282 francs in 4 weeks?
35 Which do you prefer, 24200 francs in 2 weeks, or 35431 francs in 6 weeks?
36 Which do you prefer, 24200 francs in 2 weeks, or 42872 francs in 8 weeks?
37 Which do you prefer, 8000 francs in 0 weeks, or 8000 francs in 2 weeks?
38 Which do you prefer, 8000 francs in 0 weeks, or 8000 francs in 4 weeks?
39 Which do you prefer, 20000 francs in 0 weeks, or 20000 francs in 2 weeks?
40 Which do you prefer, 20000 francs in 0 weeks, or 20000 francs in 4 weeks?

Lower-Rates Questionnaire

1 Which do you prefer, 8000 francs in 0 weeks, or 8016 francs in 2 weeks?
2 Which do you prefer, 8000 francs in 0 weeks, or 8032 francs in 4 weeks?
3 Which do you prefer, 8000 francs in 0 weeks, or 8048 francs in 6 weeks?
4 Which do you prefer, 8016 francs in 2 weeks, or 8032 francs in 4 weeks?
5 Which do you prefer, 8016 francs in 2 weeks, or 8048 francs in 6 weeks?
6 Which do you prefer, 8016 francs in 2 weeks, or 8064 francs in 8 weeks?
7 Which do you prefer, 20000 francs in 0 weeks, or 20040 francs in 2 weeks?
8 Which do you prefer, 20000 francs in 0 weeks, or 20080 francs in 4 weeks?
9 Which do you prefer, 20000 francs in 0 weeks, or 20120 francs in 6 weeks?
10 Which do you prefer, 20040 francs in 2 weeks, or 20080 francs in 4 weeks?
11 Which do you prefer, 20040 francs in 2 weeks, or 20120 francs in 6 weeks?
12 Which do you prefer, 20040 francs in 2 weeks, or 20161 francs in 8 weeks?
13 Which do you prefer, 8000 francs in 0 weeks, or 8080 francs in 2 weeks?
14 Which do you prefer, 8000 francs in 0 weeks, or 8161 francs in 4 weeks?
15 Which do you prefer, 8000 francs in 0 weeks, or 8243 francs in 6 weeks?
16 Which do you prefer, 8080 francs in 2 weeks, or 8161 francs in 4 weeks?
17 Which do you prefer, 8080 francs in 2 weeks, or 8243 francs in 6 weeks?
18 Which do you prefer, 8080 francs in 2 weeks, or 8326 francs in 8 weeks?

19 Which do you prefer, 20000 francs in 0 weeks, or 20201 francs in 2 weeks?
20 Which do you prefer, 20000 francs in 0 weeks, or 20403 francs in 4 weeks?
21 Which do you prefer, 20000 francs in 0 weeks, or 20608 francs in 6 weeks?
22 Which do you prefer, 20201 francs in 2 weeks, or 20403 francs in 4 weeks?
23 Which do you prefer, 20201 francs in 2 weeks, or 20608 francs in 6 weeks?
24 Which do you prefer, 20201 francs in 2 weeks, or 20814 francs in 8 weeks?
25 Which do you prefer, 8000 francs in 0 weeks, or 8161 francs in 2 weeks?
26 Which do you prefer, 8000 francs in 0 weeks, or 8325 francs in 4 weeks?
27 Which do you prefer, 8000 francs in 0 weeks, or 8492 francs in 6 weeks?
28 Which do you prefer, 8161 francs in 2 weeks, or 8325 francs in 4 weeks?
29 Which do you prefer, 8161 francs in 2 weeks, or 8492 francs in 6 weeks?
30 Which do you prefer, 8161 francs in 2 weeks, or 8663 francs in 8 weeks?
31 Which do you prefer, 20000 francs in 0 weeks, or 20402 francs in 2 weeks?
32 Which do you prefer, 20000 francs in 0 weeks, or 20812 francs in 4 weeks?
33 Which do you prefer, 20000 francs in 0 weeks, or 21230 francs in 6 weeks?
34 Which do you prefer, 20402 francs in 2 weeks, or 20812 francs in 4 weeks?
35 Which do you prefer, 20402 francs in 2 weeks, or 21230 francs in 6 weeks?
36 Which do you prefer, 20402 francs in 2 weeks, or 21657 francs in 8 weeks?
37 Which do you prefer, 8000 francs in 0 weeks, or 8000 francs in 2 weeks?

38 Which do you prefer, 8000 francs in 0 weeks, or 8000 francs in 4 weeks?
39 Which do you prefer, 20000 francs in 0 weeks, or 20000 francs in 2 weeks?
40 Which do you prefer, 20000 francs in 0 weeks, or 20000 francs in 4 weeks?

APPENDIX II: DEFINITIONS OF CHOICE PATTERNS

Pattern #	X today vs $X(1+r)$ in 2 weeks	X today vs $X(1+r)^2$ in 4 weeks	X today vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^2$ in 4 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^4$ in 8 weeks
	0 → earlier choice, 1 → later choice					
1	0	0	0	0	0	0
2	1	0	0	0	0	0
3	0	1	0	0	0	0
4	0	0	1	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	1	0
7	0	0	0	0	0	1
8	1	1	0	0	0	0
9	1	0	1	0	0	0
10	1	0	0	1	0	0
11	1	0	0	0	1	0
12	1	0	0	0	0	1
13	0	1	1	0	0	0
14	0	1	0	1	0	0

APPENDIX II

Continued

Pattern #	X today vs $X(1+r)$ in 2 weeks	X today vs $X(1+r)^2$ in 4 weeks	X today vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^2$ in 4 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^4$ in 8 weeks
15	0	1	0	0	1	0
16	0	1	0	0	0	1
17	0	0	1	1	0	0
18	0	0	1	0	1	0
19	0	0	1	0	0	1
20	0	0	0	1	1	0
21	0	0	0	1	0	1
22	0	0	0	0	1	1
23	1	1	1	0	0	0
24	1	1	0	1	0	0
25	1	1	0	0	1	0
26	1	1	0	0	0	1
27	1	0	1	1	0	0
28	1	0	1	0	1	0
29	1	0	1	0	0	1
30	1	0	0	1	1	0
31	1	0	0	1	0	1
32	1	0	0	0	0	1
33	0	1	1	1	0	0
34	0	1	1	0	1	0
35	0	1	1	0	0	1
36	0	1	0	1	1	0
37	0	1	0	1	0	1
38	0	1	0	0	1	1
39	0	0	1	1	1	0

APPENDIX II
Continued

Pattern #	X today vs $X(1+r)$ in 2 weeks	X today vs $X(1+r)^2$ in 4 weeks	X today vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^2$ in 4 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^3$ in 6 weeks	$X(1+r)$ in 2 weeks vs $X(1+r)^4$ in 8 weeks
40	0	0	1	1	0	1
41	0	0	1	0	1	1
54	0	1	1	1	0	1
55	0	1	1	0	1	1
56	0	0	1	1	1	1
60	1	1	1	0	1	1
61	1	1	0	1	1	1
62	1	0	1	1	1	1
63	0	1	1	1	1	1
64	1	1	1	1	1	1

NOTES

1. The dollar to franc exchange rate was 1 to 1000. The large franc values were used so that all amounts to be accurately expressed as whole numbers.
2. See Appendix II for the detailed definitions of all 64 possible patterns.
3. One might also think that Pattern 19: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$ and pattern 55: $\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ should also be included, as they seem intuitively to fit the criteria for hyperbolic patterns, i.e., there is no 1 above a 0 in a column, and no 1 the left of a 0 in a column. But these patterns can be rationalized only if $k_1 = k_2 = k_3 = k_4 = r$, the compounding rate, that is, only if the discount factor is constant and exactly equal to the compounding rate. Since any of the 64 patterns can be rationalized in this way, there is little point in singling these out for consideration either.

4. We use the phrase "average discount rate" to cover the possibility that individuals may have non-constant discount rates, as in hyperbolic discounting.
5. We used the REOPROB procedure, an "Ado" procedure in *Stata* authored by Guillaume Frechette.
6. There are indicator (dummy) variables for the high base and for the medium and high compounding rates only. So, effectively, the coefficients are $b = 0$ and $r = 0$ for the low base and low compounding rate.

REFERENCES

Ainslie, G. (1975), Specious reward: a behavioral theory of impulsiveness and impulse control, *Psychological Bulletin* 82, 463–496.

Ainslie, G. (1985), Beyond microeconomics. Conflict among interests in a multiple self as a determinant of value, In Elster, J. (ed.), *The Multiple Self,* Cambridge University Press, Cambridge.

Ainslie, G. and Haendel, V. (1983), The motives of will, In E Gotthell, K. Druley, T. Skoloda and H. Waxman, (eds.), *Ethologici Aspects of Alcohol and Drug Abuse,* Charles C. Thomas, Springfield, IL.

Benzion, U., Rapoport, A. and Yagil, J. (1989), Discount rates inferred from decisions: An experimental study, *Management Science* 35, 270–284.

Fishburn, P. (1970), *Utility Theory and Decision Making,* Wiley, New York.

Fishburn, P. and Rubinstein, A. (1982), Time preference, *International Economic Review* 23, 677–694.

Fisher, I. (1930), *The Theory of Interest as Determined by the Impatience to Spend Income and Opportunity to Invest in it*, Macmillan, New York.

Hernstein, R. J. and Mazur, J. (1987), Making up our minds: A new model of economic behavior, *The Sciences,* Nov–Dec, 40–47.

Holcomb, J. and Nelson, P. S. (1992), Another experimental look at individual time preference, *Rationality and Society* 4(2), 199–220.

Horowitz, J. K. (1988), Discounting money payoffs: An experimental analysis, Working Paper, Department of Agricultural and Resource Economics, University of Maryland.

Koopmans, T. (1960), Stationary ordinary utility and impatience, *Econometrica* 28, 287–309.

Lancaster, K. (1963), An axiomatic theory of consumer time preference, *International Economic Review* 4, 221–231.

Loewenstein, G. and Prelec, D. (1992), Anomalies in intertemporal choice: Evidence and an interpretation, *The Quarterly Journal of Economics* 107(2), 573–597.

Loewenstein, G. and Prelec, D. (1993), Preference for sequences of outcomes, *Psychological Review* 100(1), 91–108.

Loewenstein, G. and Thaler, R. (1989), Anomalies: Intertemporal choice, *Journal of Economic Perspective* 3, 181–193.
Mischel, W. (1966), Theory and research on the antecedence of self-imposed delay of reward, *Progress In Experimental Personality Research* 3, 85–132.
Mischel, W. (1974), Processes in delay of gratification, In Berkowitz, L. (ed.), *Advances in Experimental Social Psychology,* Academic Press, New York.
Mischel, W. and Ebbenson, E. (1970), Attention and delay of gratification, *Journal of Personality and Social Psychology* 16, 329–337.
Rachlin, H. (1975), *Introduction to Modern Behaviorism,* Freeman, San Francisco.
Rachlin, H. (1974), Self-control, *Behaviorism* 2, 94–107.
Strotz, R. H. (1955), Myopia and inconsistency in dynamic utility maximization, *Review of Economic Studies* 8, 165–180.
Thaler, R. (1981), Some empirical evidence on dynamic inconsistency, *Economics Letters,* 8, 201–207.

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