Risk-Taking, Financial Distress and Innovation*

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Abstract

We use our numerical technique to explore the optimality of risk-taking under financial distress. In our model, cash reserves are represented by a Brownian processes that includes an innovation parameter. When this innovation parameter goes to zero, our results show that risk-taking is optimal only when distress costs are extremely high. Thus, non-innovators need a hefty penalty to optimally take risks under financial distress. As the level of innovation increases however, it becomes optimal for innovators to undertake risky investments under financial distress without hefty penalties. The implications of our analysis might partially explain the financial crisis of 2007-2009.

Keywords: Stochastic control, numerical methods, Brownian motion, financial distress, risk

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1 Introduction

Gilson (1998) says that, because of the costs associated with financial distress, managers will "attempt to reduce the variability of operating cash flows by favoring less risky investment projects" in order to hold on to their jobs. Radner and Shepp (1996) substantiate this statement. They show that when cash reserves are low, it is optimal to choose less risky investments, and as cash reserves increase it is optimal to undertake investments of increasing risk. The reality however, is that when cash reserves are low, managers will undertake risky investments. One story involves Fred Smith, the FedEx CEO who – worried about making payments for day-to-day operations – took some money from the firms coffers to a casino in Las Vegas[1].

Clearly this sort of behavior is suboptimal to the firm. By needlessly exposing it to unnecessary risks, the manager is endangering the future of the company. We ask the question though, is it ever *optimal* to take risks when under financial distress? And, further, what conditions allow for this optimality?

With the objective of maximizing dividends, we find that it is indeed optimal for the manager to choose a risky investment when financial distress costs are considerably high for a non-innovative firm[2]. As the level of innovation increases, we find that the distress costs need not be prohibitively high for optimal risk-taking under financial distress. The uniqueness of an innovative firms products allows it to undertake greater risks to exit distress, whereas a non-innovative firm will only be forced to take risks when punished heftily under distress.

In our framework, we extend Radner and Shepp (1996) and Chen et al. (2010) to create a model of the firm that includes financial distress, that also accounts for its level of innovation and we solve it using our numerical technique. This also enables us to allow for the firm to invest in more than one investment project, which is not possible in Chen et al. (2010) where the authors create the following mixed arithmetic-geometric Brownian motion process:

$$dX_t = X^{\theta}\left(\mu_i dt + \sigma_i dW_t\right) - dZ_t, \text{where } 0 < \theta < 1 \tag{1}$$

Here, dX_t is the instantaneous increment of the cash reserve coming in at time t, μ_i is the expected rate of increase of the reserves, and σ_i is the contribution to net revenue of W_t, a standard Brownian motion or Wiener process, and $i = 1, \ldots, n$. When $\theta = 0$ or $\theta = 1$, the process becomes arithmetic or geometric Brownian motion, respectively[3]. The authors in Chen et al. (2010) call θ the *capital productivity parameter*. We however, interpret θ as an *innovation parameter*. The greater the value of θ, the greater the level of innovation to the firm. Ceteris paribus, a larger θ means a more innovative firm, and because this firm produces a unique product, this will result in a larger incremental change on X_t.

[1] *Bloomberg Businessweek*, September 20, 2004.

[2] About 20% to 60% of the risk-free rate.

[3] For $\theta = 0$, the model becomes the Radner-Shepp model.

In our model, we include an exogenous financial distress threshold below which the manager incurs a penalty in the form of reduced drift. These penalties are representative of any number of costs associated with financial distress, including extra interest charged on loans, the unavailability of commercial paper, a reduction of the accounts receivable period or simply business lost because of a reduced reputation. Purnanandam (2008) has defined them as coming from three sources:

1. The loss of customers, suppliers and employees.
2. Financial penalties imposed by missing debt payments.
3. The costs of foregoing positive NPV projects due to the increased costs of external financing.

Our objective is to maximize expected, discounted dividends over all time; our objective function is given by:

$$V(x) = \sup_{(\mu_{i,t},\sigma_{i,t}),Z_t} \mathrm{E}_x \int_0^{\infty} e^{-rt} dZ_t, \text{where } x = X_0 \tag{2}$$

We include financial distress costs into the mixed arithmetic-geometric Brownian process, that includes the innovation parameter, θ. The firm experiences the same level of risk, σ_i, but experiences financial distress costs in terms of a lowered return, i.e., $\mu_i' < \mu_i, i = 1, \ldots, n$.

$$dX_t = \begin{cases} X^{\theta}\left(\mu_i' dt + \sigma_i dW_t - dZ_t\right) & \text{if } 0 \leq X_t \leq \delta \\ X^{\theta}\left(\mu_i dt + \sigma_i dW_t - dZ_t\right) & \text{if } X_t > \delta \end{cases} \tag{3}$$

All the terms here have the same meaning as equation (1); δ is the exogenously given financial distress threshold, and μ_i' is the reduced return ($\mu_i' < \mu_i$, for $X_t \leq \delta$). The return is lowered as a cost to the firm for allowing its reserves to go below the distress threshold. Also, the innovation parameter, $0 \leq \theta < 1$. Note that when $\theta = 0$, the process becomes an arithmetic Brownian motion.

In this framework, the manager must choose from a set of available (μ_i, σ_i) pairs, in order to maximize dividends, dZ_t. The manager must decide the optimal dividend policy, and control firm value using the portfolios available. The optimal solution involves the manager giving out dividends incrementally and instantaneously whenever the cash reserve goes above an implicit threshold[4], a. Furthermore, it is optimal to switch between successively higher volatility/drift ratios as the reserve increases to a.

[4]This type of policy is known as a 'barrier policy', see Harrison (1985). The cash reserves are bound between two non-reflective barriers, since the firm ceases to exist if $X_t = 0$.

2 Methodology

Our methodology involves using Itô's Lemma, discretization of the continuous-time problem and minimization using linear programming.

We first create a process $Y(t)$ defined as follows:

$$Y_t = \overline{V}(X_t)e^{-rt} + \int_0^t e^{-rs} dZ_s \tag{4}$$

Y_t can be viewed as a total revenue process at some time, $t \geq 0$, which consists of our guess $(\overline{V}(X_t))$ as to what the profit-maximizing dividends will be into the future, discounted back to time, t; and the sum (i.e., integral) of all dividends earned from time 0 up to that same time, t.

Our methodology is based on the following lemma from Radner and Shepp (1996):

Lemma 1. *If the process Y_t is a supermartingale, then a guess, $\overline{V}(X_t)$, to the actual solution, $V(x)$, of the problem $\sup E_x \int_0^\infty e^{-rt} dZ_t$ will be greater than or equal to the actual solution.*

Proof. If Y_t is a supermartingale (an expectation-decreasing process) we have that

$$\mathrm{E}Y_\infty \leq \mathrm{E}Y_0$$

Furthermore, given that $\overline{V}(0) = 0$ (since the firm ceases to exist when $X_t = 0, \forall t$), and from (4), we have that:

$$\mathrm{E}_x Y_\infty = \mathrm{E}_x \int_0^\infty e^{-rs} dZ_s = V(x) \leq \mathrm{E}_x Y_0 = \overline{V}(x)$$

where $x = X_0$ and $\overline{V}(x)$ is a guess to the optimal solution. $\square$

Since $V(x) \leq \overline{V}(x)$ from Lemma 1, this would imply that the least upper-bound on the set of all guesses is the optimal solution. Thus, the guess must be minimized, and this is done using linear programming. But to solve the problem using linear programming, we must first translate the problem to one with explicit constraints, and then convert from continuous-time to discrete-time.

Lemma 1 rests on the fact that the process Y_t is an expectation-decreasing process. Re-stated, this means that each expected increment of Y_t should be nonpositive.

$$\mathrm{E}dY_t \leq 0 \tag{5}$$

The conditions under which (5) holds are explicit constraints to the linear programming problem. To that end, we solve for dY_t using Itô's Lemma, to get, for $i = 1, \ldots, n$:

$$\mathrm{E}\left(dY_t | X_t = x\right) = \begin{cases} e^{-rt}\left((1 - \bar{V}_X)dZ_t + (-r\bar{V} + \mu_i'\bar{V}_X + \frac{1}{2}\sigma_i^2\bar{V}_{XX})dt\right) & \text{if } 0 \leq X_t \leq \delta \\ e^{-rt}\left((1 - \bar{V}_X)dZ_t + (-r\bar{V} + \mu_i\bar{V}_X + \frac{1}{2}\sigma_i^2\bar{V}_{XX})dt\right) & \text{if } X_t > \delta \end{cases} \tag{6}$$

And since $dZ_t \geq 0$ (dividends must always be nonnegative), (6) gives us the conditions under which the process Y_t is a supermartingale (i.e., $\mathrm{E}dY_t \leq 0$). Specifically[5]:

$$
\begin{aligned}
M\bar{V} &= (1 - \bar{V}_X) \leq 0 \\
L\bar{V} &= \begin{cases} -r\bar{V} + \mu_i'\bar{V}_X + \frac{1}{2}\sigma_i^2\bar{V}_{XX} \leq 0 & \text{if } 0 \leq X_t \leq \delta \\ -r\bar{V} + \mu_i\bar{V}_X + \frac{1}{2}\sigma_i^2\bar{V}_{XX} \leq 0 & \text{if } X_t > \delta \end{cases} \\
\bar{V}(0) &= 0
\end{aligned}
$$

Since we use a numerical method, we minimize on a finite grid designed to approximate all possible values of the cash reserves, X_t. Also we have that $0 < X_t < a$ (the dividends threshold), i.e., the cash reserves are bound between two non-reflective barriers, and we can restrict our grid to be between 0 and $a + \epsilon$, where $\epsilon > 0$. Since the dividends threshold is a part of the solution, a certain amount of trial-and-error is needed to estimate a.

For the discretization process, we discretize the variable x over some suitable interval $x \in [0, x_{max}]$, where $x = jh$, with $j = 1, 2, \ldots, n_0$ and $h = x_{max}/n_0$. The $O(h)$ and $O(h^2)$ accurate approximations to the derivatives are:

$$
\begin{aligned}
\overline{V}(x_j) &= \overline{V}_j \\
\overline{V}_x(x_j) &= \frac{\overline{V}_{j+1} - \overline{V}_{j-1}}{2h} = \frac{\overline{V}_{j+2} - \overline{V}_j}{2h} \\
\overline{V}_{xx}(x_j) &= \frac{\overline{V}_{j+1} - 2\overline{V}_j + \overline{V}_{j-1}}{h^2} = \frac{\overline{V}_{j+2} - 2\overline{V}_{j+1} + \overline{V}_j}{h^2}
\end{aligned}
$$

Here, of course, $\overline{V}_x$ is the first derivative, and $\overline{V}_{xx}$ is the second derivative, with respect to x. By imposing the linear constraints at n_0 interior points, we get a total of $n_0(n+1)+1$ constraints and $n_0 + 2$ unknown variables, $\overline{V}_i, i = 1, 2, \ldots, n_0$. In general terms, the finite-dimensional problem is stated as:

$$
\begin{aligned}
&\min c^T v \\
&\text{s. t.} \\
&Av \leq b \\
&v_1 = 0
\end{aligned}
$$

where v is the unknown vector of length $n_0 + 2$, A is a $n_0(n+1)+1 \times (n_0+2)$ matrix, b is a vector of length $n_0(n+1)+1$ and c is a vector of length $n_0 + 2$. We kept $c_i = 1, \forall i$.

All output from the computational method described here, when compared with models with known solutions resulted in errors that were in the range of 10^{-4} or lower, going down to 10^{-6}.

[5]The last condition is from Lemma 1.

3 Results

We restrict our analysis to the $n = 2$ case, i.e., for when there are two (μ_i, σ_i) pairs. This is for clarity of results and convenience of execution.

3.1 Non-Innovative Firms

For non-innovative firms, the innovation parameter, $\theta = 0$, and the process collapses into arithmetic Brownian motion.

We find that when the distress-drifts ($\mu_i', i = 1, 2$) are around 20% and 60% of the risk free rate, for $i = 1, 2$, respectively[6], it is optimal for the manager to invest in the high-risk portfolio under financial distress, when $x < \delta$. When functioning outside of financial distress, $x > \delta$, the manager behaves conservatively for low cash reserves, and progressively undertakes riskier investments as cash reserves increase, until the dividend threshold, a, is reached.

3.1.1 Analytical Solution

We work to get a closed-form solution to compare with our numerical results, and we find that the results match up nicely with a mean error in the 10^{-4} range.

The technique to solve for an analytical solution involves guessing that $L_i[\overline{V}] = 0$ when it is optimal to use (μ_i, σ_i) and $L_i'[\overline{V}] = 0$ when it is optimal to use (μ_i', σ_i). Also, $M\overline{V}(x) = 0$ for $x > a$, the dividends threshold. Specifically,

$$\begin{array}{rcll} -r\overline{V} + \mu_i'\overline{V}_X + \frac{1}{2}\sigma_i^2\overline{V}_{XX} & = & 0 & \text{for } 0 \leq x \leq \delta \text{ and } i = 1, \ldots, n-1 \\ -r\overline{V} + \mu_n'\overline{V}_X + \frac{1}{2}\sigma_n^2\overline{V}_{XX} & = & 0 & \text{for } 0 \leq x \leq \delta \text{ and } i = n \\ -r\overline{V} + \mu_i\overline{V}_X + \frac{1}{2}\sigma_i^2\overline{V}_{XX} & = & 0 & \text{for } \delta \leq x \leq a \text{ and } i = 1, \ldots, n-1 \\ -r\overline{V} + \mu_n\overline{V}_X + \frac{1}{2}\sigma_n^2\overline{V}_{XX} & = & 0 & \text{for } x \leq \delta \text{ and } i = n \\ \left(1 - \overline{V}_X\right) & = & 0 & \text{for } a \leq x \end{array} \tag{7}$$

Solving these second-order, linear ordinary differential equations (ODEs) gives us the explicit functional form of the solution as follows:

$$V(x) = \begin{cases} A_i'e^{\alpha_i' x} + B_i'e^{\beta_i' x} & \text{for } 0 \leq x \leq \delta \text{ and } i = 1, \ldots, n-1 \\ A_n'e^{\alpha_n' x} + B_n'e^{\beta_n' x} & \text{for } 0 \leq x \leq \delta \text{ and } i = n \\ A_ie^{\alpha_i x} + B_ie^{\beta_i x} & \text{for } 0 \leq x \leq \delta \text{ and } i = 1, \ldots, n-1 \\ A_ne^{\alpha_n x} + B_ne^{\beta_n x} & \text{for } x \leq \delta \text{ and } i = n \\ x + \xi & \text{for } a \leq x \end{cases} \tag{8}$$

[6] They are around 33% and 83% of their respective non-distress drifts.

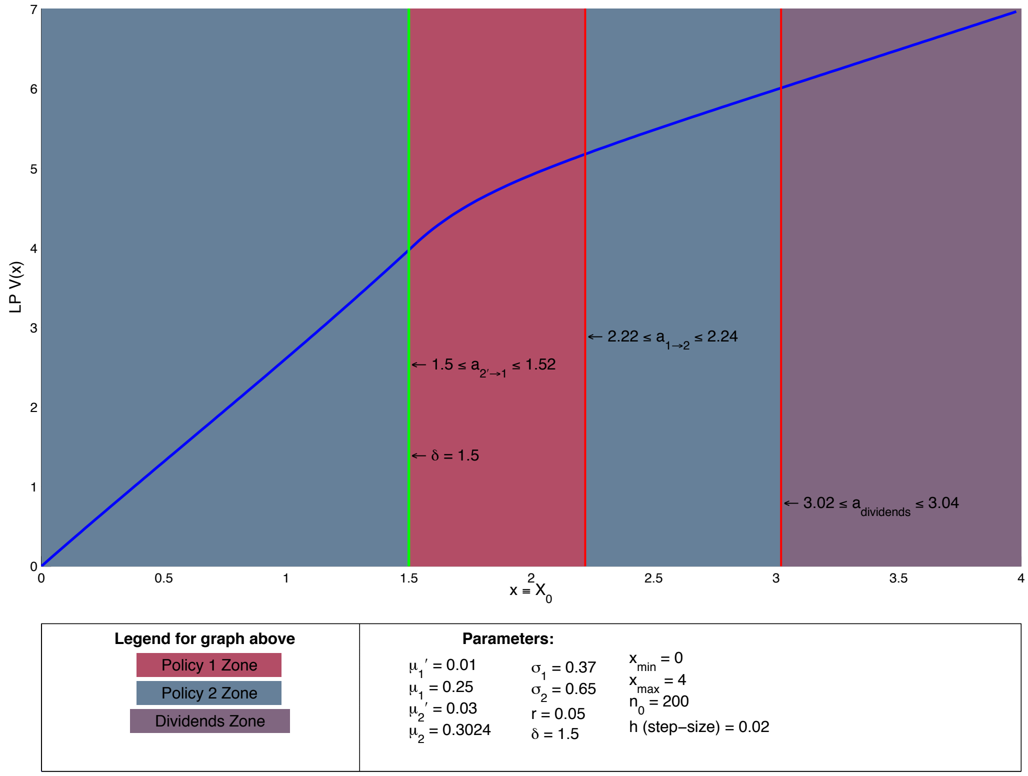

Figure 1: The manager of a non-innovative firm behaves aggressively below the financial distress threshold and behaves "normally" above it - choosing strategies that increase in risk with cash reserves. We have that $\mu_1' = \frac{1}{5}r$ and $\mu_2' = \frac{3}{5}r$ (r is the risk-free rate). Thus, for non-innovative firms, the manager must be punished aggressively to undertake optimal risk-taking under financial distress.

where all of the α and β terms are the positive and negative quadratic roots, respectively, of:

$$-\frac{1}{2}\gamma^2\sigma^2 + \gamma\mu - r = 0 \tag{9}$$

Equation (9) is the general representation of the characteristic equation of the second-order, linear ODE, where μ and σ can be replaced by adding appropriate subscripts and/or superscripts. The A and B terms are constants that are the consequence of solving the second-order, linear ODEs. These can be determined using the smooth-fit heuristic, which adds an extra degree of smoothness to $\overline{V}$ at the boundaries, thus making it easier to calculate the analytical solution. The constant ξ can also be calculated similarly.

We next calculate the analytical solution for the $n = 1$ case, which can be determined simply by solving the equations we get from (8). Using that $V(0) = 0$, we have that $A_0 = -B_0$. From this, and the smooth-fit heuristic, we can solve the set of five simultaneous equations to get our five unknowns, which are the constants A_0, A_1, B_1, a, and ξ. In doing so, we find that:

$$\begin{aligned}
A_0 &= \frac{d_1}{c_1}e^{\alpha_1(\delta-a)} + \frac{d_1}{c_1}e^{\beta_1(\delta-a)} \\
A_1 &= d_1 e^{-\alpha_1 a} \\
B_1 &= d_2 e^{-\beta_1 a} \\
a &= \frac{d_3}{\alpha_1 - \beta_1} \\
\xi &= A_1 e^{\alpha_1 a} + B_1 e^{\beta_1 a} - a
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
c_1 &= e^{\alpha_0 \delta} - e^{\beta_0 \delta} \\
c_2 &= \alpha_0 e^{\alpha_0 \delta} - \beta_0 e^{\beta_0 \delta} \\
d_1 &= \frac{-\beta_1}{\alpha_1(\alpha_1 - \beta_1)} \\
d_2 &= \frac{\alpha_1}{\beta_1(\alpha_1 - \beta_1)} \\
d_3 &= \ln\left\{\frac{e^{\alpha_1\delta}\left\{(d_1/c_1) - (\alpha_1 d_1/c_2)\right\}}{e^{\beta_1\delta}\left\{(\beta_1 d_2/c_2) - (d_2/c_1)\right\}}\right\}
\end{aligned}$$

Here, α_n and β_n are the characteristic roots of the quadratic equation: $\frac{1}{2}\gamma^2\sigma_n^2 + \gamma\mu_n - r = 0$, from the characteristic equations for the second-order, linear ODEs in equation (7). Thus, we have our analytical solution for the case of one policy, i.e., one $\mu - \sigma$ pair or when $n = 1$. When compared, our numerical solution matches up nicely with this analytical solution, and has a mean error in the 10^{-4} range.

This result gives support to our numerical technique, but with only one investment strategy to choose from, we cannot show risk-taking under financial distress. We thus move on to when θ is nonzero, i.e., innovative firms.

3.2 Innovative Firms

For innovative firms, where $0 < \theta < 1$, we find that distress costs do not have to be as low as they were for non-innovative firms for the manager to undertake risky investments optimally under financial distress. Specifically, for the $n = 2$ case, it is optimal to use the risky policy under financial distress as $\theta \to 1$, when $\mu_1' = \frac{1}{2}\mu_1$ and $\mu_2' = \frac{2}{5}\mu_2$. Above the distress threshold, when $x > \delta$, optimal managerial behavior is as seen in Radner and Shepp (1996), with investment policy riskiness increasing as the cash reserves increase.

It is optimal for the manager to undertake risky investments under financial distress as the innovation parameter increases, i.e., as the firm gets more innovative. Specifically, as θ gets

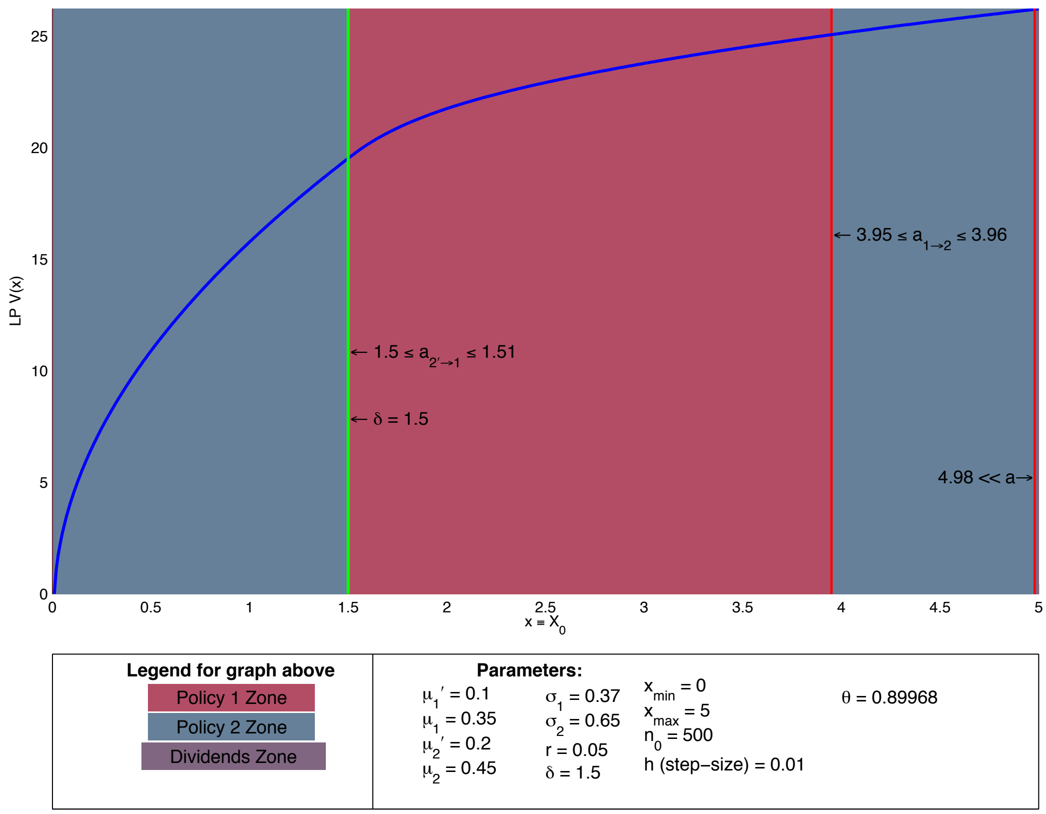

Figure 2: The manager of an innovative firm optimally undertakes a risky investment to get out of financial distress, and θ is as given above up to 6 decimal points. For the same set of parameters, we see "normal" behavior by the manager both, above and below the financial distress threshold. Note that we have restricted the state-space up to $x_{max} = 5$ for easier viewing. The actual dividends threshold, a is much larger, and explodes with increasing θ. As shown in Radner and Shepp (1996), for $\theta = 1$, $V(x) = \infty$.

closer to, but not equal to, one. Once θ gets large enough, the uniqueness of the firm's products allows the manager to undertake risky investments since the cash reserve increments (dX_t) get larger. Note that for $\theta = 1$, Radner and Shepp (1996) show that the optimal solution $V(x) = x$ when $\mu \leq r$ and $V(x) = \infty$ when $\mu > r$. It is unlikely that $\mu \leq r$, since drifts should be greater than the risk-free rate, and $V(x) = \infty$ does not make any sense. Thus, the results for $\theta = 1$ are uninteresting.

To solve analytically for innovative firms with financial distress costs is difficult. The addition of financial distress costs and increasing the available policies to $n = 2$, complicates the problem considerably and the same technique used earlier for arithmetic Brownian motion cannot be used here. We do not have the problem reduce to second-order, linear ordinary differential equations to be solved. Thus, this is left for future research.

4 Conclusion and Implications

The finance literature (Megginson, 1976, for example) states that it is both leverage and financial distress costs that cause managers to indulge in suboptimal risk-taking for the firm. We find that managers can still behave aggressively without leverage and, furthermore, that it is optimal to firm value that they do so.

Through our analysis we have two main findings. Firstly, that risk-taking by managers of firms in financial distress is optimal under certain conditions. Secondly, amongst the many factors that make risk-taking optimal are the presence of financial distress costs as well as the level of innovation in the firm.

One principal cause for the current financial crisis has to do with the mis-pricing of credit. If credit is over-priced, it becomes expensive. It becomes particularly expensive for a firm under financial distress. Since, for a non-innovative firm, this would result in prohibitively high distress costs (particularly for a firm heavily dependent on credit), this would make risk-taking optimal, and would result in managers exposing the firm, and the larger economy, to unnecessary risks. As a result, policymakers should consider calibrating subsidies and tariffs imposed on innovative versus non-innovative firms – particularly those firms under financial distress – and move towards a more dichotomous policy.

A logical step for future research would be an empirical examination of risk-taking by leaders of innovative and non-innovative firms under financial distress.

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A Innovative Firms With $n = 2$ and Without Financial Distress

When solving the Chen et al. (2010) model using our numerical technique (not done in closed form by the authors), we find a concave $V(x)$, with the low-risk policy being used, followed by the high-risk policy, which is used once cash reserves increase to a given level.

What is interesting is the the optimal switching point - the point at which it is optimal to switch from Policy 1 to Policy 2 - actually *decreases* for increases in θ. Table 1, below, outlines various thresholds for changing values of θ for three sets of parameters. While the policy-switching point (shown as $a_{1\to 2}$) decreases as θ increases, the dividends threshold, a, increases with θ[7]. This may go to explain the risk-taking behavior by the manager when under financial distress for higher levels of θ, which does not exist for lower levels of θ.

Table 1: The relationship between θ, the policy-switching point ($a_{1\to 2}$) and the dividends threshold (a), for three given sets of parameter values.

	Set 1[a]		**Set 2**[b]		**Set 3**[c]	
θ	$\mathbf{a_{1\to 2}}$	**a**	$\mathbf{a_{1\to 2}}$	**a**	$\mathbf{a_{1\to 2}}$	**a**
0	0.56	1.96	1.16	2.64	0.52	1.28
0.1	0.52	2.16	1.12	3.08	0.48	1.68
0.2	0.44	2.48	1.08	4.4	0.4	3.52
0.3	0.4	3.2	1.04	8.52	0.32	7.32
0.4	0.32	5.12	0.96	18.92	0.2	16.16
0.5	0.24	10.28	0.84	19.96	0.04	19.96
0.6	0.16	19.96	0.68	19.96	0.04	19.96
0.7	0.08	19.96	0.48	19.96	0.04	19.96
0.8	0.04	19.96	0.24	19.96	0.04	19.96
0.9	0.04	19.96	0.04	19.96	0.04	19.96

[a] $\mu_1 = .15, \mu_2 = .3, \sigma_1 = .3, \sigma_2 = .5, r = .05$
[b] $\mu_1 = .6, \mu_2 = .7, \sigma_1 = .65, \sigma_2 = .75, r = .05$
[c] $\mu_1 = .45, \mu_2 = .65, \sigma_1 = .3, \sigma_2 = .5, r = .05$

B Innovative Firms With $n = 2$ and Financial Distress

Interestingly, in this case we do not see a decrease in the post-distress optimal switching point (call this $a_{1\to 2}$), but we do see a decrease in the *pre*-distress optimal switching point (call this

[7] For this table, we used $x_{max} = 20$, $n_0 = 500$, and $h = 0.04$. Values going up to 19.96 (one step-size smaller than x_{max}) mean that they fall outside the state-space. This is case with the dividends threshold for Set 1, and is consistent with the results in Radner and Shepp (1996) where it is proved that for $\theta = 1$, $V(x) = \infty$ for $\mu > r$. Similarly the smallest step-size, 0.04, is equivalently understood to be zero, as is the case with the policy-switching point for Set 3.

$a_{1'\to 2'}$), as θ increases[8]. Table 2, below, displays the same information as Table 1, above. However, there is an additional parameter here - δ. We show how the pre- and post-distress switching points change as both, δ and θ change.

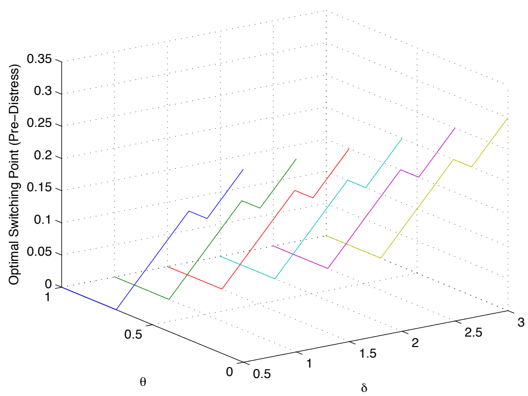
We also show a graphical representation of Table 2, in Figures 3a, 3b and 3c. These show the relationship of δ and θ versus the *pre*-distress switching point ($a_{1'\to 2'}$), the *post*-distress switching point ($a_{1\to 2}$) and the dividends threshold (a), respectively.

Table 2: The relationship between δ and θ versus the pre-distress policy-switching point ($a_{1'\to 2'}$); the post-distress policy-switching point ($a_{1\to 2}$); and the dividends threshold (a) for a given set of parameter values.

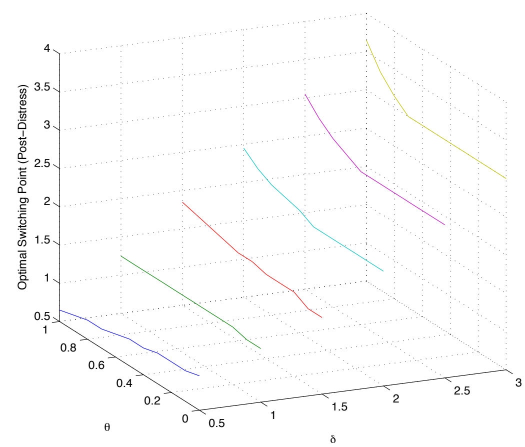
	$\delta = \mathbf{0.5}$			$\delta = \mathbf{1}$			$\delta = \mathbf{1.5}$		
θ	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$
0	0.3	0.95	2.4	0.3	1.2	2.65	0.3	1.5	2.95
0.1	0.25	0.9	2.65	0.25	1.2	2.95	0.25	1.5	3.3
0.2	0.2	0.9	3.35	0.2	1.25	3.6	0.2	1.6	3.9
0.3	0.2	0.9	19.95	0.2	1.25	5.45	0.2	1.6	12.6
0.4	0.15	0.85	11	0.15	1.25	11	0.15	1.6	11
0.5	0.1	0.85	19.95	0.1	1.25	19.95	0.1	1.65	19.95
0.6	0.05	0.8	19.95	0.05	1.25	19.95	0.05	1.65	19.95
0.7	0	0.75	19.95	0	1.25	19.95	0	1.7	19.95
0.8	0	0.75	19.95	0	1.25	19.95	0	1.75	19.95
0.9	0	0.7	19.95	0	1.25	19.95	0	1.8	19.95
1	0	0.65	19.95	0	1.25	19.95	0	1.85	19.95
	$\delta = \mathbf{2}$			$\delta = \mathbf{2.5}$			$\delta = \mathbf{3}$		
θ	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$	$\mathbf{a_{1'\to 2'}}$	$\mathbf{a_{1\to 2}}$	$\mathbf{a}$
0	0.3	2	3.35	0.3	2.5	3.75	0.3	3	4.25
0.1	0.25	2	3.7	0.25	2.5	4.1	0.25	3	4.55
0.2	0.2	2	4.25	0.2	2.5	4.65	0.2	3	5.1
0.3	0.2	2	5.75	0.2	2.5	6	0.2	3	6.3
0.4	0.15	2	11	0.15	2.5	11	0.15	3	11
0.5	0.1	2	19.95	0.1	2.5	19.95	0.1	3	19.95
0.6	0.05	2.1	19.95	0.05	2.5	19.95	0.05	3	19.95
0.7	0	2.15	19.95	0	2.6	19.95	0	3	19.95
0.8	0	2.2	19.95	0	2.7	19.95	0	3.15	19.95
0.9	0	2.3	19.95	0	2.85	19.95	0	3.35	19.95
1	0	2.45	19.95	0	3.05	19.95	0	3.65	19.95

$\mu_1' = 0.1, \mu_2' = 0.2, \mu_1 = 0.3, \mu_2 = 0.5, \sigma_1 = 0.4, \sigma_2 = 0.5, r = 0.05$

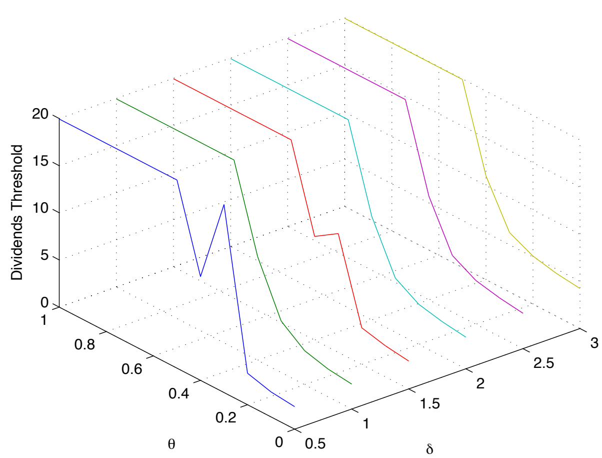
[8]To clarify, the pre-distress switching point is the point at which it is optimal to switch from one policy to the next, for those policies that are used under financial distress. Specifically, (μ_1', σ_1) and (μ_2', σ_2). The post-distress switching point is the optimal switching point to stop using (μ_1, σ_1) and start using (μ_2, σ_2).



(a) δ, θ vs. $a_{1'\to 2'}$



(b) δ, θ vs. $a_{1\to 2}$



(c) δ, θ vs. a

Figure 3: The relationship between δ and θ versus the *pre*-distress switching threshold ($a_{1'\to 2'}$); the *post*-distress switching point ($a_{1\to 2}$); and the dividends threshold (a).